MAT205 Lecture 1 Homework

Problem 1. From Groups to Simple Groups

a. Prove that if $H \leq G$ and $[G : H] = 2$, then $H \trianglelefteq G$.

b. Let $|G| = p$, where p is prime. Prove that G is cyclic.

c. Prove that every finite abelian simple group is isomorphic to $\mathbb{Z}_p$ for some prime p.

d. Explain why part (c) does not say that every finite simple group is abelian.

Problem 2. How Homomorphisms Compress Structure

a. Let $f : G \to H$ be a group homomorphism. Prove that $\ker(f) \trianglelefteq G$.

b. If $f : G \to H$ is surjective and G is abelian, prove that H is abelian.

c. Show by example that the converse is false.

d. Find a non-abelian group with an abelian quotient, and explain what structure is being forgotten in the quotient.